# Lightning electromagnetic field generated by grounding electrode considering soil ionization\*

ZENG Rong\*\*, HE Jinliang, ZHANG Bo and GAO Yanqing

(State Key Laboratory of Power Systems, Department of Electrical Engineering, Tsinghua University, Beijing 100084, China)

Received January 12, 2006; revised March 24, 2006

Abstract A circuit model with lumped time-variable parameter is proposed to calculate the transient characteristic of grounding electrode under lightning current, which takes into consideration the dynamic and nonlinear effect of soil ionization around the grounding electrode. The ionization phenomena in the soil are simulated by means of time-variable parameters under appropriate conditions. The generated electromagnetic field in the air is analyzed by using electrical dipole theory and image theory when the lightning current flows into the grounding electrode. The influence of soil ionization on the electromagnetic field is investigated.

Keywords: electromagnetic field, grounding electrode, soil ionization, transient characteristic.

The lightning impulse performance of grounding systems plays an important role in the safe and reliable operation of power system. The lightning protection effects of substations or transmission lines are related to the impulse characteristics of grounding systems.

When lightning strikes a substation or transmission lines, high currents will flow into the grounding system and dissipate into the soil. The electromagnetic field generated by such high currents may cause severe damage to the electronic equipment and power apparatus in the power system by radiation. Therefore, how to accurately model and predict the transient characteristic of a grounding system under the consideration of ionization is very important.

As already proved by many researches, the characteristic of grounding electrodes subject to high impulse current is dramatically different from that at low frequency with low current injection, because the inductive behavior becomes more important than resistive behavior, and the large current can generate complicated soil ionization surrounding the grounding conductors, which makes the transient characteristic typically non-linear.

Many papers have pointed out that high current, such as lightning current, will cause soil ionization, which tends to result in a lower resistance and a lower transient voltage on the electrode. There have been many papers on the performance analysis of grounding systems<sup>[1,2]</sup>. And several models have been proposed to analyze and predict the behavior of grounding electrodes when they are excited by impulse currents. However, the dynamic and non-linear ionization phenomenon is often omitted because the complicated physical phenomenon is too difficult to model.

In this study, a circuit model with lumped timevariable parameter is proposed to calculate the transient characteristic of a grounding electrode under lightning current. When the lightning current flows into the grounding electrode, the generated electromagnetic field near the grounding electrode is also analyzed by using electrical dipole theory and image theory.

# 1 Equivalent circuit model of a grounding system

When high impulse current is put on a grounding system, the transient electromagnetic fields around the grounding conductors will be generated in the nearby soil as

$$E = J\rho, \tag{1}$$

where E is the electric field strength in the soil,  $\rho$  is the soil resistivity, and J is the current density in the soil.

<sup>\*</sup> Supported by National Natural Science Foundation of China (Grant No. 50407002) and the Major State Basic Research Development Program of China (Grant No. 2004CB217906)

<sup>\*\*</sup> To whom correspondence should be addressed. E-mail: zengrong@tsinghua.edu.cn

When the electric field strength surrounding the grounding conductor exceeds the critical value of the electrical field strength ( $E_{\rm c}$ ) of soil, the breakdown of the soil will occur around the conductor. This will change part of the soil from an insulator to a conductor. The fall of potential in the area of soil ionization is often omitted, because the resistivity of the area of soil ionization is approximately zero. The radius of the area of soil ionization surrounding the grounding electrode can be considered to be the equivalent radius of the electrode during the transient process.

It can be found that the shape of the ionized zone of the soil around a grounding conductor is not cylinder but pyramidal. Considering the complexity of the mathematical model, we propose a simple model shown in Fig. 1, in which the conductor is represented by a set of cylindrical zones. Obviously, this assumption is reasonable. In Fig. 1  $r_i$  is the equivalent radius for the i-th segment, which is time-variable when an impulse current is injected into the grounding conductor; and a is the radius of the metal conductor. A grounding conductor under impulse current can be considered to be a distributed network as shown in Fig. 2, which is composed of inductance, capacitance, conductance and resistance. This model was described in detail in Ref. [3].

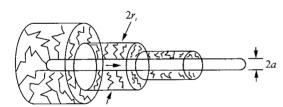


Fig. 1. The pyramidal simulation model of the ionized electrode.

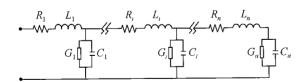


Fig. 2. The simulation circuit of electrode with distributed parameters.

The unit length resistance, inductance, capacitance, and conductance of a grounding conductor are given in Refs. [4] and [5]. The capacitance of a grounding conductor with a length of l and a radius of a in an infinite medium is

$$C_{i}(a) = \frac{2\pi\varepsilon l}{\frac{a}{l} + \ln\frac{l + \sqrt{l^{2} + a^{2}}}{a} - \sqrt{1 + \left(\frac{a}{l}\right)^{2}}}.$$
(2)

When the conductor is buried in a depth of h, the capacitance is obtained by assuming two conductors with distance 2h in an infinite medium, which is based on image theory. Therefore, the capacitance of a buried grounding conductor equals

$$C = C_{l}(a) + C_{l}(2h - a). {3}$$

When the capacitance has been designated, the inductance and conductance can also be determined  $bv^{[6]}$ 

$$G = \frac{C}{\varepsilon \rho}.$$
 (4)

In order to simulate the ionization effects, the equivalent radius of the grounding conductor is considered time-varying under appropriate conditions. The electrical parameters of the conductor tied to the diameter of the conductor (including capacitance C and conductance G) are also time-varying according to Eqs. (3) and (4).

As is well known, the electric field intensity on the boundary of the ionized zone is the critical value of soil breakdown. The equivalent radius for each segment can be obtained by

$$J_{i} = \frac{E_{c}}{\rho} = \frac{\Delta i_{i}}{2\pi r_{i} \Delta l},$$
 (5)

where  $J_i$  is the current density leaked by the *i*-th segment,  $\Delta i_i$  is the current leaked into earth from the *i*-th segment, and  $\Delta l$  is the length of the *i*-th segment.

In this study, the parameters mentioned above changed at each time step in calculation. Based on the circuit theory approach, this model will be much easier than most of the previous models. The impulse response of a complex grounding electrode when considering the nonlinearity soil ionization is investigated.

# 2 Transient electromagnetic field analysis

#### 2.1 Computation methodologies

The lightning current dissipates into the soil while traveling along the grounding conductor. Simultaneously, very strong electromagnetic fields are generated both in air and in soil.

Any grounding conductors in the soil may be regarded as being made up of a large number of short electrical dipoles. The total radiated electromagnetic field is the sum of all the contributions of each constituent dipole. Each dipole is short enough that the

current can be considered to be a constant along its length. The grounding conductors are divided into many segments, which are short enough to be considered to be Hertzian dipoles.

The vector potential A of a horizontal dipole in an infinite conductive medium can be derived from the Maxwell equations<sup>[7]</sup>:

$$A(r) = \frac{\mu_0}{4\pi} \int_0^\infty \frac{\lambda}{u} e^{-u|z-z'|} J_0(\lambda \rho) d\lambda \alpha, \quad (6)$$

where

$$u_1 = \sqrt{\lambda^2 - k_1^2},$$

$$\rho = [(x - x')^2 + (y - y')^2]^{1/2}.$$

A dipole in a two-layer media is illustrated in a Cartesian coordinates system in Fig. 3. The upper medium is Region ① (z>0), and the lower medium is Region ② (z<0). There is a horizontal electrical dipole at the position r' in Region 2.

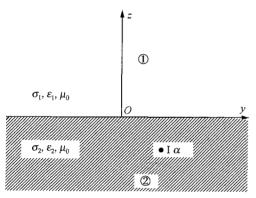


Fig. 3. Electrical dipole in two-layer media.

The vector potential **A** in Regions ① and ② can be obtained by using the electrical dipole theory and the image theory. The following cases can be considered for the position of the source and observation point.

### 1) Observation point in Region ① (z>0).

When we calculate the electromagnetic field in Region ①, we assume the whole medium as an infinite one with parameters  $(\sigma_1, \ \epsilon_1, \ \mu_0)$ . The electric field can be evaluated as the direct radiated field due to the modified image current source I', whose amplitude value and direction are determined by the boundary conditions.

#### 2) Observation point in Region 2 (z < 0),

We assume the whole medium as an infinite one

with ( $\sigma_2$ ,  $\varepsilon_2$ ,  $\mu_0$ ). The total electromagnetic field will then be the sum of the direct radiated field from the conductor section and the reflected field from the ground plane, whose amplitude value and direction are determined by the boundary conditions.

According to the fundamental analysis<sup>[7,8]</sup>, the vector potential  $\mathbf{A}$  consists of the x-component  $A_x$  and the z-component  $A_z$ :

$$\mathbf{A} = A_{r}i + A_{r}k. \tag{7}$$

Solutions in both media are connected by the boundary conditions at the interface (z = 0). On the boundary plane (z = 0), the following boundary conditions must hold:

$$\begin{cases} A_{1x} = A_{2x} \\ \frac{\partial}{\partial z} A_{1x} = \frac{\partial}{\partial z} A_{2x} \end{cases}$$
 (8)

$$\begin{cases} A_{1z} = A_{2z} \\ \frac{1}{k_1^2} \left( \frac{\partial}{\partial x} A_{1x} + \frac{\partial}{\partial z} A_{1z} \right) = \frac{1}{k_2^2} \left( \frac{\partial}{\partial x} A_{2x} + \frac{\partial}{\partial z} A_{2z} \right), \\ \text{with} \qquad k_1^2 = -j\omega\mu_0\sigma_1, \qquad k_2^2 = -j\omega\mu_0\sigma_2. \end{cases}$$
(9)

In Region ① (z > 0), the x-component of **A** has the expression

$$A_{1x} = \frac{\mu_0}{4\pi} \int_0^\infty g(\lambda) \frac{\lambda}{u_1} e^{-u_1 | z - z'|} J_0(\lambda \rho) d\lambda, (10)$$

with z' < 0,  $u_1 = \sqrt{\lambda^2 - k_1^2}$ ,  $k_1^2 = -j\omega\mu_0\sigma_1$ . And  $g(\lambda)$  is an uncertain coefficient determined by the boundary conditions. The image source is located at the position r'.

In Region ① (z > 0), the z-component of **A** has the expression of

$$A_{1z} = \frac{\mu_0}{4\pi} \cos\varphi \int_0^\infty a(\lambda) e^{-u_1 z} J_1(\lambda \rho) d\lambda, \quad (11)$$

where

$$a(\lambda) = \frac{2\lambda^2}{u_1 + u_2} \frac{k_1^2 - k_2^2}{u_1 k_2^2 + u_2 k_1^2} e^{u_2 z'}.$$
 (12)

According to the Lorentz condition

$$\nabla \cdot \mathbf{A} + \mu_0 \sigma \varphi = 0, \tag{13}$$

we can obtain

$$\varphi = -\frac{1}{\mu_0 \sigma} \nabla \cdot \mathbf{A} = \frac{\mathrm{j}\omega}{k^2} \nabla \cdot \mathbf{A}. \tag{14}$$

Once the vector potential A has been solved, the corresponding electric and magnetic fields in the air (z>0) can be obtained. The electromagnetic field

can be obtained by

$$E = -\nabla \varphi - j\omega \mathbf{A} = -j\omega \left[ \mathbf{A} + \frac{1}{k^2} \nabla (\nabla \cdot \mathbf{A}) \right], \tag{15}$$

$$\mathbf{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A} \,. \tag{16}$$

#### 2.2 Disposal of lightning current

A double exponential lightning current with  $2.6/50~\mu s$  wave shape and 20~kA amplitude is injected into the ground electrode at one end.

$$i(t) = I_{\rm m}(e^{-k_1 t} - e^{-k_2 t}),$$
 (17)  
 $a_1 = 20.938 \text{ kA}, k_1 = 0.0146 \text{ us}^{-1}, k_2 = 0.0146 \text{ us}^{-1}$ 

where  $I_{\rm m}=20.938$  kA,  $k_1=0.0146~\mu{\rm s}^{-1}$ ,  $k_2=1.883~\mu{\rm s}^{-1}$ , i(t) and t are in units of kA and  $\mu{\rm s}$ , respectively.

First, the lightning current is transformed into its frequency domain by a forward FFT operation. A finite number of currents consisting of a representative sample of the frequency spectrum are selected. The frequency domain performance of the grounding electrode can be obtained for each of the selected single frequency current source. Then the time domain behavior of the electromagnetic fields generated by the lightning transient at one point in air can be obtained by an inverse Fourier transformation of all the responses to this point.

# 2.3 Influence of soil ionization

When soil ionization is taken into consideration, the transient electromagnetic field generated by lightning current will be different from those without considering soil ionization. Fig. 4 illustrates the computation model. Here, the soil is assumed to be homogeneous with a  $100~\Omega$  m resistivity, the relative permittivity is 1, and the relative permeability is 1.

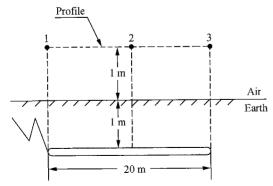


Fig. 4. Model adopted for simulation.

When a lightning current is injected in a ground-

ing electrode with 20 m in length (Fig. 4), which is buried at 1-meter depth under the earth, severe transient electric field will be generated in air. Comparing the magnitudes of electromagnetic field at points 1, 2, and 3, we can obtain some interesting results of the influence of soil ionization on transient electromagnetic field.

Fig. 5 shows the magnitudes of electric field with and without soil ionization at points 1, 2, and 3 illustrated in Fig. 4.

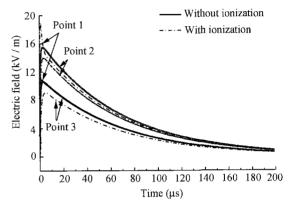


Fig. 5. The electric field at different points.

From the curves in Fig. 5, when soil ionization is taken into consideration, the electric field in air near the position of feed point will be much severer than that without soil ionization. On the other hand, the electric fields in air far from the feed point become smaller.

The curves in Fig. 6 show the maximum electric field along the profile illustrated in Fig. 4. It can be found that the maximum electric field will be at the feed point considering soil ionization instead at the middle part without considering the soil ionization. It can be explained that the current density is much higher at the feed point than that at the end point.

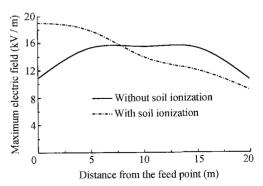


Fig. 6. Maximum electric field along the profile.

Fig. 7 shows the magnitudes of magnetic field with and without soil ionization for points 1, 2, and 3 in Fig. 4. When soil ionization is taken into consideration, the magnetic field in the three points will be a little smaller. The curves in Fig. 8 show the maximum magnetic field along the profile, the influence of considering soil ionization on magnetic field is very slight.

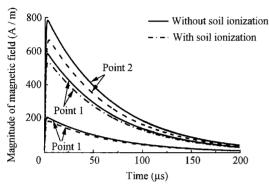


Fig. 7. Magnitude of magnetic field at different points.

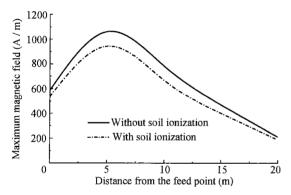


Fig. 8. Maximum magnetic field along the profile.

# 3 Conclusions

A circuit model with lumped time-variable parameter is proposed to simulate the transient characteristic of a grounding system under lightning current. The ionization phenomena around the ground electrode in the soil can be simulated by means of time-variable parameters under appropriate conditions.

When the lightning current flows into the grounding electrode, the electromagnetic field generated by the grounding electrode is analyzed by using electrical dipole theory and image theory. If the soil ionization is considered, its influence on electrical field is very significant, but its effect on the magnetic field is small. Therefore, to precisely analyze the electromagnetic environment, especially the electrical field generated by the grounding electrodes under lightning strike, the soil ionization neighboring the grounding electrode must be considered.

#### References

- Oettle E. E. A new estimation curve for predicting the impulse impedance of concentrated earth electrodes. IEEE Transaction on Power Delivery, 1988, 3(4): 2020—2029.
- 2 Leonid D. G. and Markus H. Frequency dependent and transient characteristics of substation grounding systems. IEEE Transaction on Power Delivery, 1997, 12(1): 172—178.
- 3 He J. L., Gao Y. Q., Zeng R. et al. Effective length of counterpoise wire under lightning current. IEEE Transactions on Power Delivery, 2005, 20(2): 1585—1591.
- 4 Xiong W. and Dawalibi F. Transient performance of substation grounding systems subjected to lightning and similar surge currents. IEEE Transactions on Power Delivery, 1994, 9 (3): 1412—1417.
- 5 Sunde E. D. Surge characteristics of a buried bare wire. AIEE Transactions, 1940, 59; 987—991.
- 6 Geri A. Behavior of grounding systems excited by high impulse currents; the model and its validation. IEEE Transactions on Power Delivery, 1999, 14(3); 1008—1017.
- 7 Lei Y. Z. The study of 3-dimensional eddy current field and its inverse problems in nondestructive testing. Ph. D. thesis (in Chinese), Tsinghua University, China, 1995.
- 8 Leonid D. G. and Frank E. M. Transient electromagnetic fields near large earthing systems. IEEE Transactions on Magnetics, 1996, 32(3): 1525—1528.